Equals

Realising potential in mathematics for all

for ages 3 to 18+

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Spot on with Number

Equals

Realising
potential in mathematics
for all

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Designed by Nicole Lane

Editors' Page

Time to celebrate

This edition of *Equals* celebrates two very special people:

- Alexsandra Young the winner of the Ray Gibbons Memorial Award,
- Olivia Taylor who has been awarded the Harry Hewitt Memorial Prize.

It gives us great joy to celebrate their stories and to highlight just what makes them so worthy of this recognition. By way of introduction Mark Pepper has taken the time to let us know who Harry Hewitt was and why the award was instigated in the first place. Harry, who died 20 years ago, was held in very high regard by all those involved in *Equals* and such was the impact of his life and work that an award was introduced to ensure his legacy to the SEND community would live on. The Ray Gibbons Memorial Award is the first one given as it is in recognition of the life and work of Ray Gibbons the driving force behind *Equals* for many, many years. If you know of anyone who you feel deserves recognition in this way then please get in touch next March when the nominations will open once again.

For the past year *Equals* has been part of the Maths Hubs SEND working group meetings that have been taking place in Birmingham. What a year it was and what a privilege it has been to be among so many colleagues who passionately share the same belief in the potential of all children that we at *Equals* do. Space does not permit me to share, and do justice, to what has been done over the last 12 months but the outcomes of this work will be shared via the NCETM. What we will do at *Equals* is, during the coming academic year, showcase the work and ideas that we feel are most easily shared in this format. So watch this space for news of the great work that has been taking place this year around England.

On the 29th June Pete Jarret and I supported Kirsty Behan at the London MA/ATM branch meeting whose focus this time was upon SEND. The turn out was wonderful and for the first time I felt *Equals* not only had a voice but we had something to say that was actually helping teachers and their pupils. It could be that, with your help, we can grow into something that can help SEND pupils within mathematics across the United Kingdom and beyond. It may be that the time is right for a full-scale *Equals* conference next year! If you would like to be involved then please get in touch. We are ending the year on a high and this is in large part due to the number of people who have stepped forward to share what they do in their classrooms. As an example of this I suggest you read the piece in this edition by Nicky White.

Equals is above all things a network for colleagues to support each other through the things that they do in their classrooms. Even a short account of something that worked for you and your children may be just the help another colleague is looking for. So why not put pen to paper this Summer and share something through *Equals*?

This edition of *Equals* can be split into four sections:

1. The 2019 *Equals* Awards,
2. Read and respond,
3. Sharing ideas and research and,
4. Resources.

In section one Louise Needham shares her thoughts on presenting both the Harry Hewitt Memorial Prize and the Ray Gibbons Memorial Award. I think you will agree that the personal testimonies, given in support of the two winners, serve to highlight just why Olivia Taylor and Aleksandra Young deserve to be recognised in this way. These prizes serve many purposes but their primary function is to give professional recognition to those who deserve it. Many years ago I too, like Louise, presented the Harry Hewitt Prize and I still remember the high that came with standing in front of a sea of primary faces all cheering the recipient of the award. This section begins with a reminder, by Mark Pepper, of who Harry Hewitt was and why it was felt necessary to instigate an award in his name.

Section two is a regular feature of *Equals* and the name 'Read and respond' best sums up the thinking the articles seek to provoke. This time Mark Pepper has commented upon the piece on Algebra by Tricia Strickland from the last edition. Do you agree with him? He has also taken the time to respond to the recent Cambridge investigation into anxiety in mathematics. A couple of years ago we published a special edition of *Equals* focusing upon anxiety – what are your views on the recommendations of the Cambridge Review?

Section 3 is concerned with the sharing and dissemination of any research that may be of relevance to SEND. Nicky White, Special School teacher from Newcastle and Lead Evaluator for the NCETM, got in touch to share her research on number lines. There is much to reflect and act upon in what she has written, which is the result of many years of careful research and practice. Nicky has also agreed to become part of the *Equals* team which is lovely news. Please let us know what you think of her work and also why not take the time to share your own views on vertical numbers lines; how do you use them, have they helped in your classroom and if you have tried some of the activities she suggests. Lesley Goddard has submitted a very interesting and timely research summary on Memory and Retention. I think this makes an excellent starting point for anyone who is interested in applying the research on memory to their classroom with the aim of helping their students prepare for the tougher linear examinations.

Finally section 4 provides an opportunity to publicise a resource that may be of help in the SEND classroom. Spot on With Number is a new resource and in this piece Carol Handyside explains just why you may wish to give it a try. In this section we have also included Kirsty Behan's piece on the June meeting of the London MA branch at which Pete Jarrett and I shared some activities and thoughts on supporting SEND.

An Introduction to The Harry Hewitt Memorial Prize

Mark Pepper reminds us who **Harry Hewitt** was and why *Equals* started to recognise learners in this way.

Introduction

The Harry Hewitt Memorial Prize was introduced in 1999 shortly after Harry's death and it has continued to take place in subsequent years. This is an appropriate time to explain why Harry was so special and the reasons why it is fitting that his name should be remembered.

I am heavily indebted to an appreciation of Harry written by Ray Gibbons (1999). This article perfectly described the personality and great skills of Harry which I cannot emulate. I will, however, draw on some of the points that Ray so eloquently presented.

Harry's professional career encompassed the roles of teacher, head of a special school and warden at Webber Row Teachers Centre for special educational needs. It was at Webber Row that a group of Inner London Education Authority (ILEA) mathematics advisers initiated *Struggle* – a most unusual publication as it was exclusively devoted to maths and SEN. This publication evolved into a renaming of *Equals*. From this point on references to *Struggle* also apply to *Equals*.

My first meeting with Harry

I first met Harry when a friend and I visited the Webber Row Teachers' Centre for the first time. I was immediately impressed by Harry who spent a considerable amount of time in showing us round and then responding to our questions.

Meeting Harry at a *Struggle* meeting

I next encountered Harry about a year later. I had for the first time submitted an article to *Struggle* with a request that it be considered for publication. The editor formally invited me to a meeting to discuss the article with the editorial team. Towards the end of the meeting Harry suggested to the rest of the team that I should be invited to join them. They agreed to this and I was very pleased to accept the offer. At that time I was a very inexperienced primary teacher and I joined a highly qualified and experienced team. I will always be indebted to Harry for the support and encouragement that he provided for me at that time. Over the years I gained greater experience and became increasingly confident in my contribution of articles and reviews to *Struggle*. Throughout this time I always greatly valued Harry's advice. An example of this arose when I had a fairly weak title for an article that I had written. After he had read it Harry suggested an alternative title which fitted it perfectly. Harry was never adversely critical but he consistently offered encouragement and support. He also took a close interest in my teaching career and offered helpful advice especially when things were going badly. I also valued Harry's friendship in the more general aspects of life and I have especially fond memories of going for coffee with him after *Struggle* meetings.

Harry's great strengths

Perhaps Harry's greatest strength consisted of his remarkable analytic powers which he applied both to general educational issues and in reviewing articles submitted to *Struggle*. He was never negative but would pick out the strengths of articles and make suggestions of how particular themes could be expanded. He also had a remarkable ability to defuse tensions that arose from time to time at Struggle meetings. He consistently remained calm and he possessed a quiet, dry sense of humour.

The importance of the memory of Harry

When one considers that Harry's death took place twenty years ago the vast majority of readers of *Equals* would not have had the privilege of meeting Harry or even be familiar with his name. I hope this appreciation goes some way to conveying the remarkable qualities that Harry possessed.

Mark Pepper

Reference

Gibbons, R. (1999) Harry Hewitt 1923-1999: An appreciation
Equals Vol.5 No.3

The Harry Hewitt Memorial Prize winner and The Ray Gibbons Memorial Award winner

Louise Needham was able to present both awards on our behalf – read her reports to find out what happened when she did.

Harry Hewitt Memorial Prize

Olivia attends Hollinwood Academy in Oldham. Hollinwood Academy is a state-of-the-art specialist provision for pupils from the age of 4 up to 19 whose Autistic Spectrum Condition and Social Communication Needs have become a barrier to their learning.

Olivia started at Hollinwood Academy in year 10, she proved herself to be a very able mathematician

with a determination to do well in the subject. She is an absolute delight to have in the classroom and wants to succeed in the subject. In June 2018 she sat her foundation GSCE maths paper and achieved a high grade 5. The following September she wanted the opportunity to sit the Higher GCSE paper in the November. Olivia worked extremely hard for this exam and was showing great potential when answering the higher level problem solving GCSE questions.

When the first paper arrived in November Olivia was looking forward to sitting the exam, however things took a dramatic turn when half an hour into the exam Olivia's anxieties took the better of her. Olivia suffered with extreme anxiety, to point in which the exam had to be terminated. It was heartbreaking to watch this young lady crumble before our very eyes. However, with the support of the staff team at the Hollinwood Academy, Olivia went on to sit and complete the remaining two exams. This took a shear amount of courage by Olivia, and an acceptance that it was ok to accept the coaching that the staff team offered her to complete these further two exams. Olivia found the courage to talk about her anxieties and to talk about the struggles that she was having. She developed a strong trust with the staff that worked with her, and now has some lasting relationships that will go beyond her time at Hollinwood Academy.

I am so proud to present this award to Olivia, as she has shown true courage to overcome her anxieties, this is no mean feat for anybody to do, but Olivia also had her ASD to contend with. She is a wonderful student to teach and I wish her every success for the future!

"When we received the phone call from Mrs Needham that Olivia had won an award we couldn't have been prouder but she was already a champion to us. To know that others recognise how much she achieves everyday is so wonderful.
Olivia loves Maths and it comes naturally to her but the added stresses of exams, time management and her ASD makes everything overwhelming for her. Thankfully many strategies were put in place and Mrs Needham's over whelming support to her throughout the exam process allowed Olivia to be able to continue. Olivia didn't give up. She's amazing!" - Lisa Taylor (Olivia's mother)

"What an achievement for Olivia and a testament to her own strength of character and perseverance and we are all delighted for her.

From the Dalai Lama
"Whenever there is a challenge, there is also an opportunity to face it, to demonstrate and develop our will and determination."

We think really sums up Olivia's journey and recognises her determination to face and overcome barriers now and in the future – Well done!" - Graham Quinn CEO of the New Bridge Group

"Olivia is a very polite young lady who is enthusiastic and passionate about food, art and writing. She is articulate and can hold a good conversation, particularly when she is discussing things she is passionate about such as, movies, food and books. Olivia's smile is the brightest when she is eating chocolate. Olivia is extremely talented and creative which is clearly displayed through her attention to detail. She is a fantastic writing and has aspirations

to become an author in the future. Olivia's family is very important to her, she has recently become an Auntie and loves spending time with them and going to restaurants for family meals." - Ms Bennett, Ms Ismail and Mr Valentine - Pastoral Managers at Hollinwood Academy

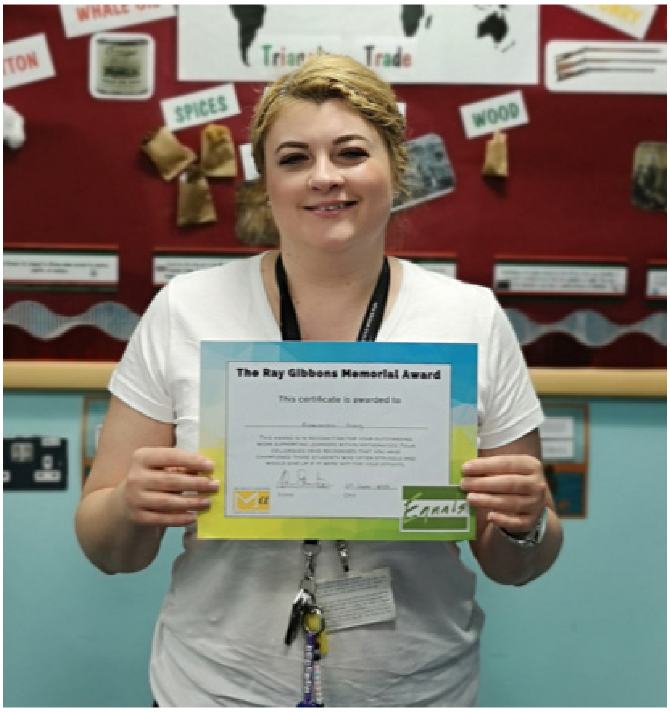
"Olivia has made significant progress since starting at Hollinwood Academy, however this has not been an easy journey. She has overcome many barriers to learning in her efforts to work outside her comfort zone. Olivia's anxieties at times have prevented her from achieving her goals; however she hasn't given up. Olivia puts a lot of pressure on herself, she sets her own goals, these are often ambitious and not without challenge because she strives to be the best she can. I am extremely proud of Olivia's developments and she should be too." - Ms Gordon Deputy Head at Hollinwood Academy

"Olivia is a truly remarkable young lady who has had to overcome many battles in educational settings. She joined Hollinwood Academy in January 2018 after attending a mainstream school. Olivia struggled in mainstream due to her ASD. She was often isolated in lessons and missed a lot of learning opportunities with her peers. Olivia is a wonderful young lady who has a natural talent in maths. All of us at Hollinwood Academy and her mum are immensely proud of Olivia for receiving this award for maths, it represents her accomplishments whilst at Hollinwood Academy. She is more than worthy of this prize and more" - Ms Bones Curriculum lead of Maths at Hollinwood Academy.

"Olivia is a bright young girl with lots of potential. She is naturally polite and enthusiastic and is very thoughtful. Her compassion and determination has allowed her to reach her goals and I am so proud of how far she has come. I have loved watching Olivia's knowledge and passion for Maths grow in the past two years and I can't wait to see what the future holds for her." - Ms Sanderson Olivia's form tutor at Hollinwood Academy.

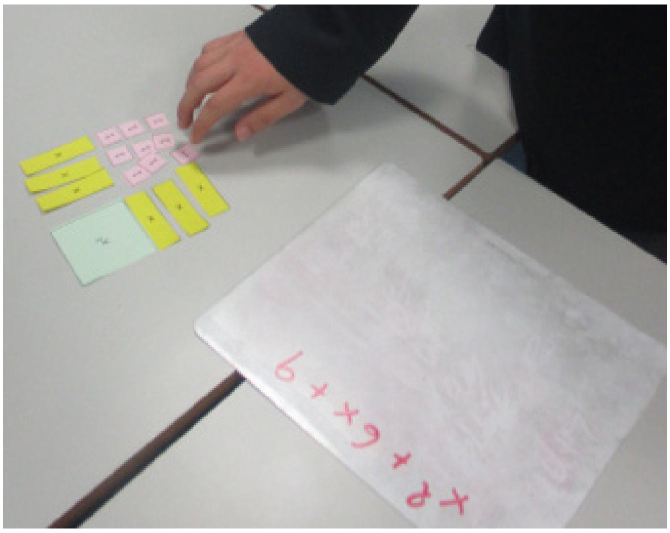
Ray Gibbons Memorial Award

This month I had the pleasure of presenting Aleksandra Young with the Ray Gibbons Mathematics Award. Ola Young is a secondary teacher at Spring Brook Academy in Oldham. The majority of the pupils at Spring Brook Academy travel from all areas of Oldham, some move from mainstream schools while others have joined us from establishments outside the Authority. Spring Brook Academy is a unique place where pupils are nurtured and developed to encourage independence and skills for life, and their social,

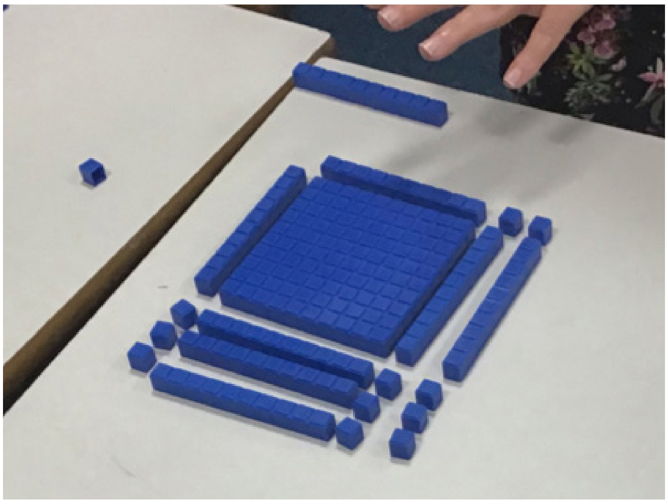


emotional and mental health needs are at the forefront of our work.

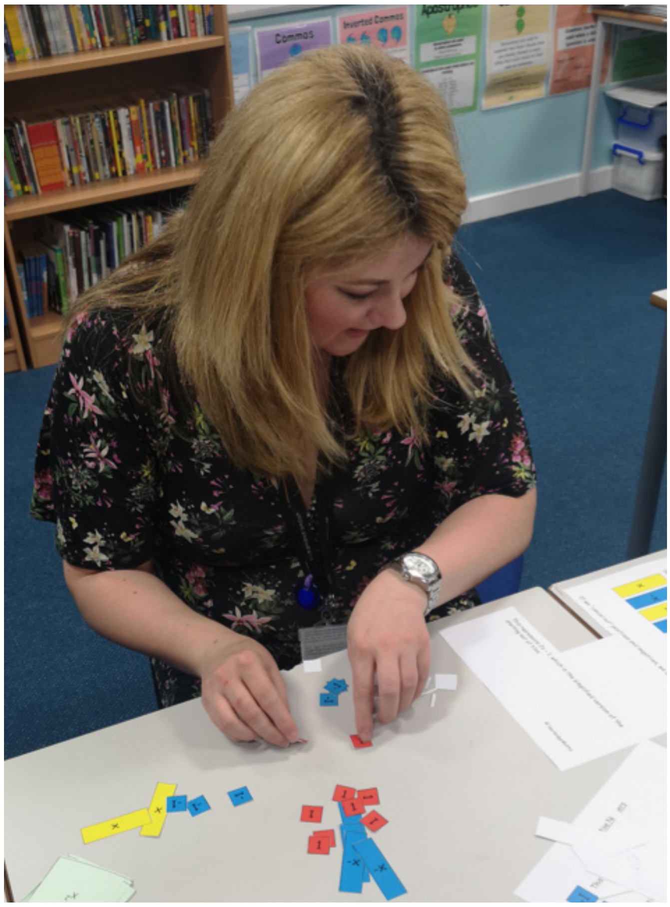
Ola is an inspirational teacher of SEN pupils. She uses the best pedagogy so that pupils are able to access mathematical concepts. The concrete-pictorial-abstract-language approach to teaching is at the heart of all her lessons. For example, in an expanding brackets lesson, Ola used the methods of long multiplication of double digits to make links to using algebra tiles, so that in future lessons pupils can make links to the abstract concept of expanding double brackets. The lesson made a difficult abstract concept accessible for all the pupils in the lesson.



Ola is not afraid to take risks when it comes to teaching. If something doesn't work the first time that she uses it, she will adapt and try again. She



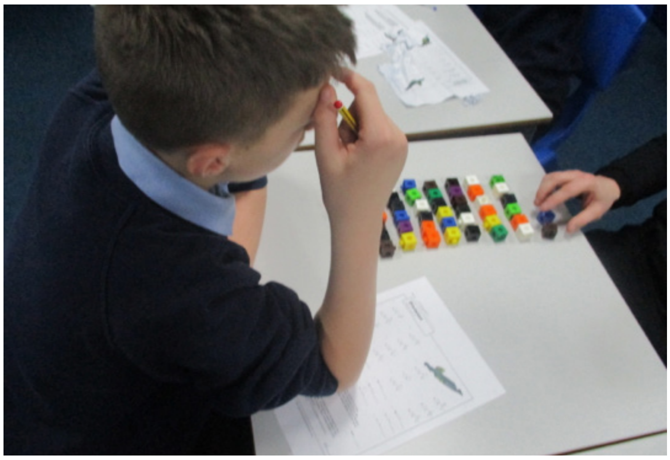
is very creative in her approaches to teaching mathematics; her lessons are inspirational. The young people that she works with are some of the most challenging and vulnerable within the education system. The young people often arrive at Spring Brook very disengaged with education, however Ola has an amazing ability to re-engage these pupils and challenge them to the point in which they succeed. Ola is a very worthy winner of this prize and I am extremely proud to present it to her.



Here are some comments from the staff at Spring Brook Academy:

"Ola is an amazing teacher across all subjects. The reason I nominated her for the maths award was around her unique ability to engage hard to reach learners. Ola has successfully engaged these learners through using a range of styles. She caters very well for their individual needs. It has been a joy popping in and out of Ola's lessons overtime to see Year 9 SEMH boys hooked on working out mathematical problems at one stage using a toy till. Ola has established positive relationships with all of the students she teaches. She really does not understand how beautifully she has crafted the art of teaching in order to get the very best from her learners. " - Miss Rodgers Head of KS3

"I have worked with Ola at our primary provision and I couldn't be more delighted that she has been recognised in this way. We have always known that Ola is an amazing teacher but it took a while for her to believe us! She is so passionate about every child deserving the very best and doesn't stop working towards this. She has developed trusting relationships with the boys in her class and this is evident in the way that they are now prepared to take risks with their learning. Well done Ola, we are really proud of you!" - Toni Thomason Head of Spring Brook Lower

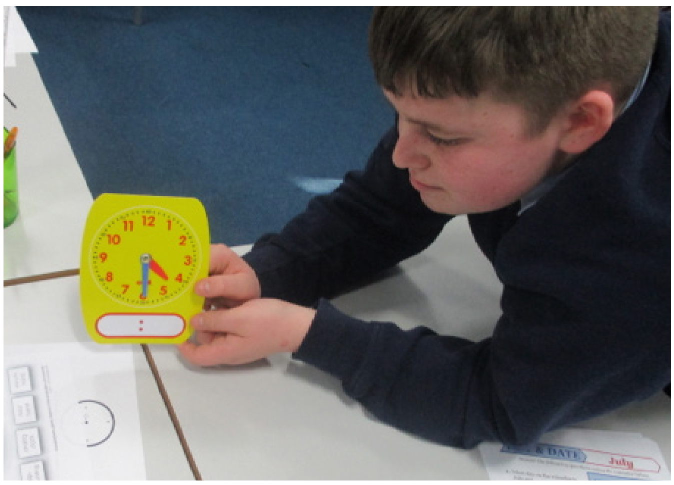


"We are absolutely delighted that Ola has picked up this award, which acknowledges the passion and dedication that Ola and indeed all of our staff demonstrate daily for all of our young people/adults, to consistently give them the very best offer within our trust.

It is just recognition for her work ethic, attitude and commitment in all of her classes – Well done Ola" - Graham Quinn CEO of the New Bridge Group

Here are some comments from the staff team at the Spring Brook Upper site:

"Ola is a great teacher and a credit to the school, with a great laugh 😀 "



“Ola is an amazing teacher and has great relationships with everyone and is a credit to Spring Brook”

‘Ola is great at differentiating her work for all learners. Helping them achieve in the best way”

“Ola is the best teacher I’ve ever worked with!”

“There is just one word for Ola and that’s AMAZING!’

“Ola teaches with a nurturing approach. A total pleasure to see how she caters for individual needs of students in her class”

“Ola is full of patience, nurture and understanding of the children. Great teacher!”

“Ola has a good understanding of her students and their needs!”

“Ola is a fantastic teacher. She is creative, inspiring and enthusiastic about everything that she teaches!”

Louise Needham

A summary of the 2019 Cambridge Review of Mathematics Anxiety

Mark Pepper responds to this review with some personal thoughts and suggestions.

Understanding Mathematics Anxiety:
Investigating the experiences of UK primary and secondary school students
University of Cambridge
Nuffield Foundation
March 2019

A study of maths anxiety is most welcome as it is a theme that has largely been neglected both within Government policy and by commentators of mathematics education. A notable exception of this trend consisted of Laurie Buxton’s seminal article *Maths Phobia* which first appeared in *Struggle* in 1992 and was subsequently reprinted in *Equals* Vol 22 No 3 in 2017.

The study was conducted by researchers from the centre for neuroscience in education from Cambridge University. It involved 2700 primary and secondary students from the UK and from Italy.

It provides a considerable amount of details of the maths anxiety experienced by the students and includes the use of one-to-one interviews in which some of the students express their feelings of despair and inadequacy.

The Main Findings

Most of the findings will be of no surprise to experienced teachers of maths. There is, however, one significantly unexpected finding in that a high proportion of middle to higher achievers experience maths anxiety. This runs counter to the widely held belief that widespread incidence of maths anxiety is confined to lower achievers. The discovery that it is also prevalent in higher achievers may encourage teachers to amend their teaching approach to take this into account.

A further significant finding based on the research in Italy was that maths anxiety is more prevalent in girls than in boys.

A further significant finding based on the research in Italy was that maths anxiety is more prevalent in girls than in boys.

Recommendations

A section of the study bears the sub-heading *recommendations* and these are disappointingly unhelpful. A general concern with them is that they are of a nebulous nature. Rather than make concrete suggestions that could easily be put into practice in a classroom setting there are vague appeals for further research and descriptive assertions as opposed to suggestions for new policies.

One of the recommendations asserts that:

"Teachers need to be conscious that individual's anxiety likely affects their maths performance."

If the authors of this study believe that teachers need to be directed to an understanding that a strong correlation exists between anxiety and maths performance then one can only conclude that the authors grossly underestimate the empathy that most teachers have with the members of their class. On a continual basis a major preoccupation of maths teachers is a desire to reduce anxiety both out of consideration for the comfort of their students and to try to improve their performance.

Another recommendation states that:

"Teachers and parents need to be conscious of the fact that their own mathematics anxiety might influence student mathematics anxiety…Hence for parents and teachers, tackling their own anxieties and belief systems in mathematics might be the first step to helping their children or students."

Of course some parents and teachers have difficulties with their own mathematics. It is highly probable that they will be acutely conscious of this. It is also extremely likely that many of them, especially the teachers, would already have gone to considerable lengths to try to remedy this. The effect of this recommendation is likely to be that the teachers would become even more demoralised. No advice is provided on how they should "tackle their own anxieties."

Of course some parents and teachers have difficulties with their own mathematics.

The effects of Government policy

Minimal consideration is made on the effect that Government policy has on the generation of maths anxiety. A brief reference to this is made in the context of the psychological condition of the students:

"In the UK little attempt is made by government policy to investigate affective problems (such as mood and anxiety) on a large scale amongst UK students."

There is also a brief allusion to National Tests (incorrectly referred to as SATs).

"Year 6 SATs were also a trigger for increased maths anxiety…"

It is disappointing that there is no further consideration of the effect of Government policy with regard to the excessive use of mandatory exams and the way in which the test results have been used to assess the performance of individual teachers. It is hardly surprising that with so much at stake teachers commonly feel obliged to put pressure on their students to achieve acceptable results. These difficulties are further exacerbated both by the pressure for teachers to make widespread use of Maths Mastery and to provide the content to meet the requirements of National Tests. This has led to a heavy dependence on the learning of number facts and the repetitive application of taught algorithms. A consequence of this is that there are few opportunities for the students to consider a variety of computational strategies or to develop logical reasoning skills. The levels of anxiety for the students would be heightened by the expectation to reproduce the mechanical devices that they have been taught in a formal exam setting. One encouraging development involves the changes that are due to take place in September 2019 regarding the inspection of schools when a new framework is due to be introduced in which there will be a shift in emphasis away from an appraisal of exam results to a consideration of issues such as the breadth of the curriculum.

It is hardly surprising that with so much at stake teachers commonly feel obliged to put pressure on their students to achieve acceptable results.

Conclusion

The strength of this report involves the impressive use of qualitative research which has resulted in the collection of data which effectively conveys the unpleasant experiences of maths anxiety that has affected many of the students. A major weakness is the failure to use this information to present suggestions aimed at reducing levels of maths anxiety. There is an absence of suggestions such as making concrete resources available, the use of self- assessment, increased use of I.T. and making maths lessons more stimulating and enjoyable with a removal of the current emphasis on preparation for exams that test a narrow range of formal skills.

Mark Pepper

N.B.The views expressed here are those of the author and not necessarily shared by the editorial team of *Equals.*

A response to Tricia Stickland's piece on teaching equations

Mark Pepper shares his thoughts on the work presented by **Tricia** in the last edition of *Equals*.

The central theme of team teaching by a maths teacher and an SEN teacher is to be welcomed as it is a topic that has received little previous attention. The fact that it is made clear that the two teachers act on an equal basis, as opposed to the SEN teacher being subservient to the maths teacher, is also to be welcomed.

There are, however, considerable difficulties with this dissertation. The opening sentence of the Absract states:

"Secondary students with disabilities are expected to achieve within the high school mathematics curriculum to the same level as their non-disabled peers, although an achievement gap exists."

This sentence consists of a contradiction in terms. If the "students with disabilities" have achieved at the same level as their "non-disabled peers" then there cannot be an achievement gap.

A more fundamental difficulty arises from the total absence of a definition within this context of "student with disabilities". The reader can only assume that this means students with special educational needs (SEN). There is, of course, a wide range of different conditions within SEN. These include behavioural disorders as well as a wide range of physical disabilities including a visual impairment and a hearing impairment. As none of these conditions on their own should have any bearing on mathematical attainment, then this group can be discounted. By a process of elimination it is reasonable to assume that the disability refers to a learning disability. There is, of course, a wide range of ability within those students classified as having a learning disability. These range from severe learning difficulties (SLD) to moderate learning difficulties (MLD). If the assumption is made that the disability in this context, exclusively consists of a moderate learning difficulty then a different problem arises. In my experience it is inconceivable that a student with MLD would be able to solve relatively complex equations. Furthermore the assumption that the provision of small group or even individual teaching by an SEN teacher would remedy this situation lacks credibility.

Conclusion

It is essential to include an introduction in which the term "students with disabilities" is removed. It would appear that in reality the group consists of mainstream students who are achieving at a lower level than the majority of their classmates. If this were to be clearly specified the dissertation would then provide a credible teaching strategy.

Mark Pepper

Can Vertical Number Lines support pupils with poor working memory?

Nicky White kindly shares her research on number lines.

On the diagnosis of one member of my class with Fetal Alcohol Spectrum Disorder (FASD) I was given an advisory document (Blackburn et al 2010) for teachers. This document led to a further personal exploration of FASD and maths instruction. Re-occurring suggestions of slower pace, visual aids, concrete resources, activity based learning and quiet working space were already a part of our maths classroom. However, one suggested tool which was not common place in my classroom was the vertical number line (Blackburn, Kable et al 2007).

one suggested tool which was not common place in my classroom was the vertical number line

The umbrella term FASD is used where there are lasting effects to a child after exposure to alcohol pre-nataly. FASD is thought to affect approximately 1 in 100 live births, being the biggest non-genetic cause of learning disability (Samson et al 1997). Goswami and Bryant feel that due to the negative effects of alcohol on the parietal lobe when developing in the womb children often experience serious problems with Mathematics. (Goswami and Bryant 2007).

Kable suggested that the use of a vertical number line to assist understanding that when adding the numbers go up and down for subtraction (Kable et al, 2007). Kable also felt the physicality of the tool would assist in working memory problems, a common feature in FASD. Many other pupils I was working with have diagnoses that encompass visual and spatial processing difficulties as well as difficulties in working memory so I thought this could possibly be a tool to assist others as well.

The use of number lines and tracks are commonly seen in British primary classrooms (Bottle 2005). A useful tool in order to assist pupils to 'visualise' our number system (Cockburn 1999) but not always an obvious choice (Bruno and Cabrera 2006). Both types can support pupils to gain mental mathematical imagery when used in both play and work (Skinner and Stevens 2013). An understanding of order and pattern in our number system, which can simply be supported with a number line, can help with the development of mental calculations (Mosley 2001:4).

A number track is a series of squares with an individual number placed in each square usually starting with 1 to 10 or 20. A tool to help pupils understand the cardinality and order of numbers up to 20 (NECTM). Skemp (1991:140) and Bottle (2005:98) feel the number track is a physical tool leading to the use and understanding of a number line, a conceptual tool, a line with numbers placed on markers. The number track has a defined beginning and end whereas a number line can go on infinitely. The placement of a number on a number-line is on a marker rather than a space as with the number-track. Space between equally spaced marked integrals making the link with measurement and scale. Skemp argues the number line is more sophisticated and can assist fractions, negative numbers and decimal fluidity. Number lines and tracks are believed to have a positive effect on children's mathematics (Griffin 2014). Links have been found between performance in mental arithmetic and math test scores with children's ability to make mental number line representation (Ramani and Siegler, 2008). This has also been seen in children with mathematical learning disabilities (Geary et al 2008:277). 'Experience' of using number lines will assist in children being able to develop a good understanding of multiplication, especially when they can talk about the 'jumps' and relate to patterns (Anghileri 2008:49). They can be used as a tool for modelling and encourage mathematical talk (Parrish 2014), enhancing the understanding of our number system (Askew 2012).

These results do indicate that the brain is doing some sort of categorising, but does it really support a horizontal number line?

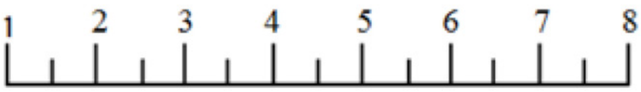


Figure 1. Horizontal number line

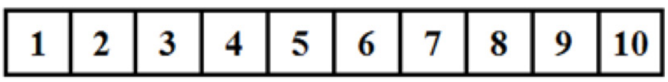


Figure 2. Horizontal number track

During a learning walk around our school, common practice favoured a horizontal direction with little focus on vertical number lines except when working on temperatures. This, I have also noticed being reflected in classroom displays of number lines and tracks in other schools I have visited. In Mosley's (2001) book 'Using Number Lines' only horizontal versions of number lines are used.

Do humans possess an internal mental number line (MNL)?

Dehaene's work on a 'Number Sense Theory' (Dehaene 1997) where he proposes that humans are naturally disposed to be able to categorise things due to their numerosity and proposed a visuospatial perception that orders small numbers to the left and larger to the right. In his research participants choose left or right responses whether a number was odd or even. Response times were quicker for larger numbers on the right and smaller on the left, even when hands were crossed over (Dehaene et al 1993). He also found that odd numbers produced quicker responses on the left and even on the right. This result was known as the Spatial-Numerical Association Response Codes effect (SNARC effect; Dehaene, Bossini and Giraud, 1993). These results do indicate that the brain is doing some sort of categorising, but does it really support a horizontal number line? Especially as when he found participants who were brought up in a culture that wrote right-to-left the SNARC effect was reversed. These effects were replicated by other studies (Fischer, Castel, Dodd, & Pratt, 2003). When only offered a left or right response it predetermines that results will be horizontal, neglecting any other mapping of numbers including vertical.

Gobel (2015) found when comparing both adults and children from Britain and Hong Kong a preference of direction for counting strongly related to cultural reading direction. However young children in his study showed a preference for counting vertically from the bottom up. This she feels is due to experience with the physical world and magnitude. Smaller equating to low and the larger the number to be higher, for example going up in a lift or the number of stairs climbed. Ebersbach (2015) found that preschool children did show a preference for number order from left to right as opposed to right to left when using a numerical estimation test, but did not test the vertical line. These children were thought to be before the onset of reading, I would argue that my children at three could not read but had already been influenced in the direction tracking from left to right by enjoying books from a baby. This would support a constructivist's view of mathematics, that our mathematical knowledge is shaped through childhood into adulthood by cultural inventions.

Looking further back in history, Galton's article 'Visualised Numerals' in 1880 found that some people showed very precise spatial locations for numbers. This could be classed as synaesthesia where there are 'joined sensations' (Cytowic & Eagleman 2011:1). On seeing, hearing, tasting, touching or smelling it is paired with another sense, the most common being colour and words. Galton named people's view of numbers as "natural lines of thought" (Galton, 1880). The 'number forms' Galton found ranged from being a similar line which sometimes moved in orientation, to other forms which were more elaborate and were comparable to grid forms. Others started off in a circular motion for the first 12 digits before going in a more linear fashion. The circular form with 12 digits is obviously comparable to the image of a clock which has long historical standing. Other features were also present, colours, brightness and presentation of different planes.

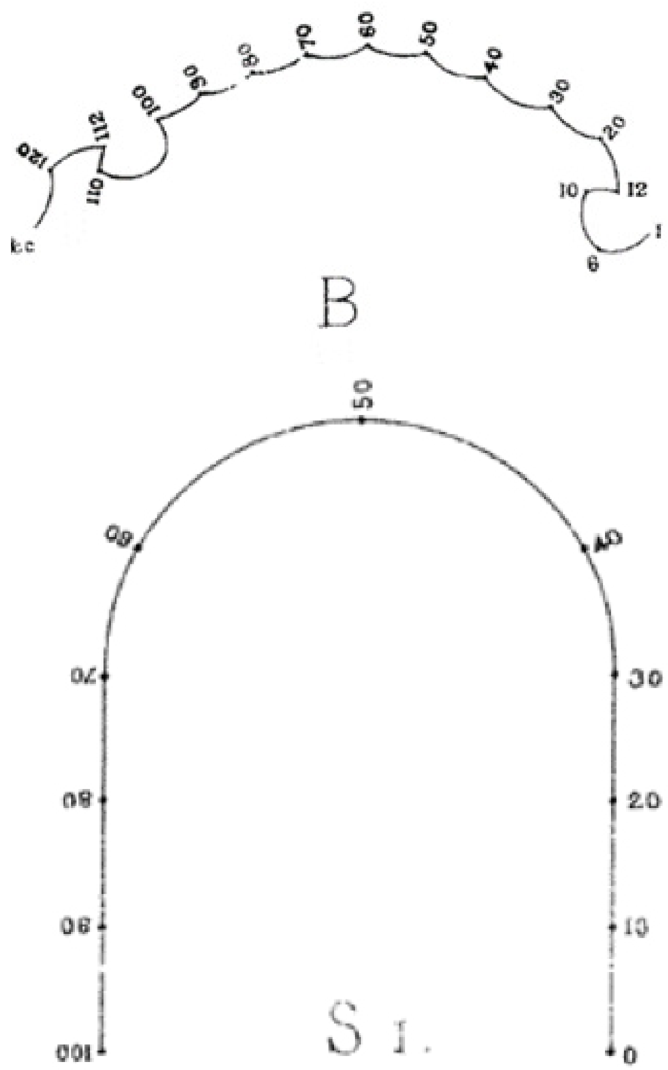


Figure 3. Examples taken from Gaulton, Visualised numerals 1880

Xavier Seron (Seron et al., 1992) found out of 194 subjects 14.6% had a mental number line. His participants went under a series of test to ensure their lines were not fabrications but consistently represented. Butterworth (1999) does not feel there is any link between those who report in having a MNL and mathematical ability. He found that all that representations have in common is that they use numerals not words and majority mark decade boundaries, the first 10 standout as being different. Out of 15 number lines reported, Serion found that 15 had left-to-right orientation with only 1 right-to-left, 10 had a down-up orientation and 1 had the reverse. 2 subjects had a clock like orientation like Galton one clockwise and one anti clockwise.

Dehaele et al (2008) postulated that perhaps humans have an intuition for mapping numbers into space but only some have conscious recall of their inner number line, but that these internal visuo-spatial constructions are developed, influenced by cultural experiences like reading and experience of the world around us like clocks.

The role of language.

It is possible that due to the complexities of the English language this is more of a language problem. Kable's reasoning for Vertical number lines making sense when linking it to language '*adding* results in numbers going up and *subtracting* results in numbers going down' (Kable 2007). Molina (2013) examines the complexity of the English language and how the conception meaning is often confused by the language of mathematical instructions. Our language also contains homonyms, words that sound the same but are very different (some and sum). Molina points out how we use ambiguous language; drawing upon the example asking pupils to 'add something up' when we expect them to put the answer vertically below the question.

Dunkirk and Shire (1991:80) studied young children's understanding of spatial terms applied to numbers. Four and Five year olds were asked to write high and low numbers on a rectangular card. Typically the four year olds applied spatial knowledge writing them up at the top and bottom of the card and the five year olds used numerical knowledge. Dunkirk (1991) acknowledges that this misunderstanding is normally an issue in 'middle childhood' however they did find similar confusion when older pupils were asked about ordinal numbers when a team 'moved up' from 8th by three spaces.

Typically the four year olds applied spatial knowledge writing them up at the top and bottom of the card and the five year olds used numerical knowledge.

Lexical ambiguity obviously impacts greatly on pupils with language difficulties. Working memory needs to juggle information in the short term memory and retrieve information from the long term memory. If a pupil's understanding of 'up' in mathematical terms is not secure then the demands on the short term memory to juggle that and numbers would be problematic. A vertical number line or track could possibly give pupils a visual that assists with a calculation.

I decided to carry out some research to explore whether vertical number-lines would support pupils in a Special School setting. If vertical number lines would assist and alleviate some pressure on the working memory aiding understanding or enable computation. Nunes and Bryant (1996:18) state that 'we can only think mathematically with concepts that mean something..'. As a large number of pupils in the school, especially in the primary, were working at foundation level I also encompassed the use of vertical number-tracks, often seen as a pre-curser to the number-line (Skemp 1991; Cockburn 1999). I felt an achievable goal in my research was to find out the perspective of pupils and teachers about vertical number-lines and tracks and if they were in fact beneficial as suggested by Kable and in which areas of maths. The 'action research' took place over a six week period. I collected data through a questionnaire given to all maths teaching staff pre and post the trial. During the research I wrote a 'diary' of my experiences in class. From the questionnaire I discovered some common themes which I also cross referenced in my research journal. At the end of the period I carried out interviews. All staff were included in a 'staff training session', so were able to support pupils in the research period from an informed position.

The evidence from the questionnaires, research diary and interviews show that for some pupils the vertical version of tracks did help develop pupils understanding.

I interviewed four members of staff who between them encompassed the full academic range covered from p scales to entry level/GCSE, from pupils from key-stage 1 to Key-stage 4. I wrote five initial questions shaped by the questionnaires and in the interview I asked more which came from their answers

Research period

A period of six weeks was set aside for the research. This scale was settled on, to allow time to explore the use of vertical numberlines and tracks in more than one area of maths for each class. This of course would not allow all possible areas of numberline usage to be explored by each group, but a longer research may lead to less focus on the tool/ manipulative. All pupils who took part in numeracy classes where numbering-lines or tracks were being utilised were included in this exploration. However, working with children with complex needs, it was important no pressure was put on them to use this method and they could revert to the more common place horizontal version.

Due to the nature and complex needs our pupils have, I was not completely satisfied that I could ensure they would understand the research process which is needed for informed consent. I felt the best way would be to find out about pupil's response through teaching staff. Due to high staff student ratios it is perhaps easier for staff to observe pupil's response and interaction to a tool/manipulative than in other settings. However this would always carry a risk of adults perceived child's view.

The evidence from the questionnaires, research diary and interviews show that for some pupils the vertical version of tracks did help develop pupils understanding. The areas where this was reported frequently and details of 'learning' events seen, was in adding, subtracting and counting. All these are widely agreed to be appropriate uses of the Number track (Skemp: Bottle: Thomson)

The use of the vertical number line was less convincing, although there were 'some' students who it seemed to be of assistance there were also students who did not find them useful or even workable and some who were 'resistant' to using it. Resistance was expected due to the nature of our pupils and the reluctance to try 'new' ways. If, as Dehaene (1993) found, the SNARC effect becomes fixed in mid childhood then a new line model would challenge the more able student's internal number line, which would have been influenced by reading, earlier math's instruction and the world around them, for example rulers. I know for myself trying to use a vertical empty number line did not come naturally. This could indicate why one member of staff said that 'Just horizontal makes more sense.'

Common themes were seen in the data relating to track or line success; movement, visual and language. The positive response for learning when pupils were moving along large vertical forms could indicate pupils learning possibilities are increased when they are enjoying and engaging in an activity. There are thoughts that the brain is in a better position to learn when there is body movement (Ratey, Hagerman 2008). There are programmes where maths and movement are intertwined, the movement not always linked to an explanation of a maths concept (mathdance, Math&movement).

Common themes were seen in the data relating to track or line success; movement, visual and language.

It could be more than just movement of physical exercise, that the movements help the understanding of the concept. Jumping 'up and down a ladder' Masoum et al 2013 saw gains in mathematical understanding including abstract concepts when using drama. The floor based interaction allowed children to play going up and down the number system.

Another theme that was mentioned was the visual element to the track. This element was to be expected as number lines are often seen as a useful tool to 'visualise' our number system (Cockburn 1999).Half the data mentioned visual benefits of the vertical version, especially the number track, seeming to support some pupil's understanding far more than a horizontal version which supported their reasoning. Common response pupils could see the numbers going 'up' and 'down'. However, the more I think about this statement the more complex I think it is. If the vertical number line or track gives aid to visual spatial processing that would suggest an innate vertical number line. Whilst I am still sceptical about the horizontal SNARC effect (Dehaene, Bossini and Giraud, 1993), and it not being dramatically affected by experiences, if numbers are a human construction then surely an internal number line must be too. If the numbers are 'discovered' they must at some point have an element of construction as they are matched with Arabic representations to show the number boundaries. Unsurprisingly this research has not shown otherwise. I feel the research did show for some pupils the vertical number line and track were of a support, more so the track than the line. I think the visual aspect is important but because the 'lexical ambiguity' (Durkin1991a:80) barrier is removed. This may indeed help those with visuo-spatial processing and working memory issues as there is not internal conflict necessary to jump the abstract mathematical language whilst also having to work with numbers. This explanation would help explain that the vertical track appeared to be of greater assistance. When pupil's mathematical skills developed and they move onto number lines there is also a deeper understanding of common mathematical language.

The use of number lines and tracks would appear to be used throughout the school for a range of calculations. The research has shown that pupils actively engage with tracks and lines and that they appear to be a useful tool in mathematical development. The data seems to suggest that the younger less mathematical able pupils were more receptive to a vertical version, with some respondents remarking on pupils not noticing a difference. Older pupils seemed less able to adapt to the vertical form, perhaps because it was 'different' as one interviewee suggested. Lack of concrete understanding of how to use number lines could have also played a part.

All interviewees felt that the vertical version was something to pursue in their class and at the very least be on offer for pupils to use if they wish. Three interview respondents felt if pupils progressed through the school with practical experience of the vertical number track and line a difference may be seen in it assisting older pupils which may help them develop concrete understanding of calculations using a number line.

This research project was carried out in a small special school setting so the findings need to be taken with this in mind. Due to this a large data collection was not possible.

Nicky White

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Memory and Retention

Lesley Goddard, in the first of two pieces, presents a very timely summary of the research supporting this very important issue and one which many schools are taking more and more seriously.

Matt Bromley says: “Learning is the acquisition of knowledge and skills and their application at a later time and in a range of contexts” which I’d summarise as “acquire, retain and apply”.

Doug Rohrer 2009 says “The effects of forgetting are often neglected by learning theorists, but acquisition has little utility unless material is retained. Indeed, although poor performance on standardized achievement tests is often attributed to the absence of acquisition, forgetting may often be the culprit.” which I’d summarise as “if learners don’t retain their learning, it doesn’t matter how well they acquired and applied that learning at the point of teaching”

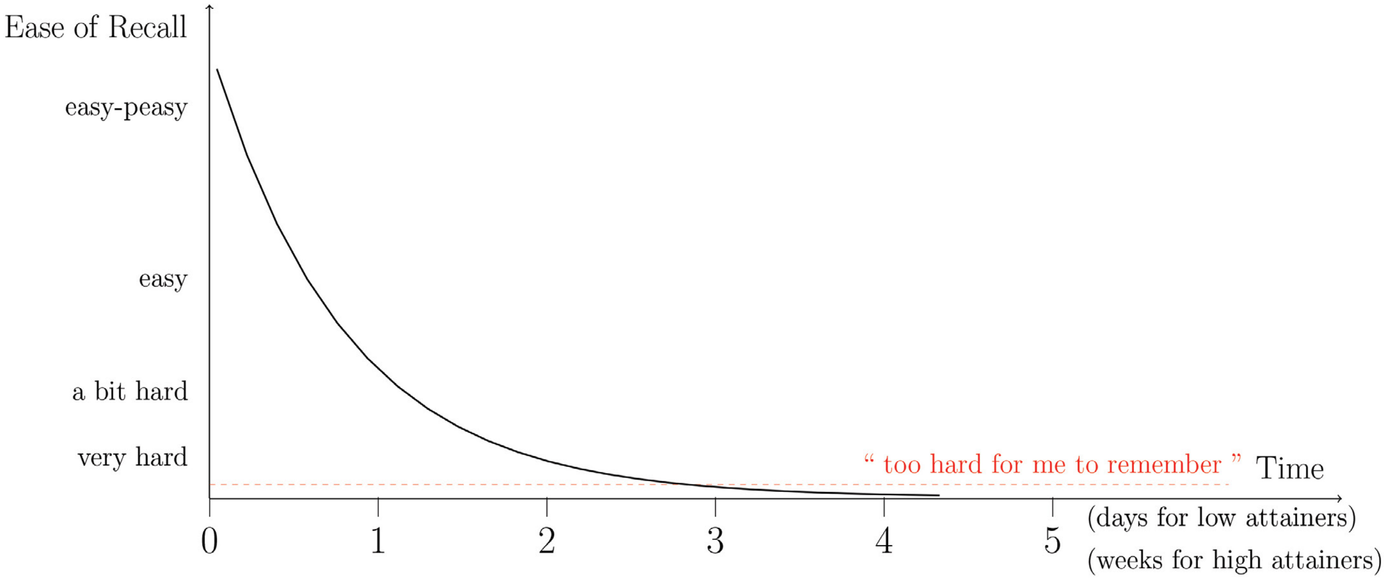
“if learners don’t retain their learning, it doesn’t matter how well they acquired and applied that learning at the point of teaching”

For many years I’d bought into the theory that multiple good lessons would enable me to raise the attainment of “my low attaining learners”. I worked hard, and was relatively successful in planning and teaching lessons that (i) activated links to prior learning, (ii) were well differentiated and (iii) encouraged active learning using carefully chosen or created resources. In the short term my lessons were successful, but in the long term, not so much. Most low attaining learners soon forgot what they appeared to have learned. I realised that unless I could somehow enable them to overcome their forgetting, my teaching would continue to be largely wasted.

In this article I’ll share what I consider the most useful academic research which I’ve learned about on my journey to answer the question “what can I, as a teacher, do about the forgetting of low attaining maths learners?” I’ll summarise what I’ve learned from a teacher’s perspective and share how this research links to my experience: 25 years as a maths teacher, and 6 years of working towards a “more effective way for low attaining learners to learn maths”.

Let’s start with a little history.

Ebbinghaus’ 1885 forgetting curve tells us: If we learn something and don’t use it, over time we will forget it. None of us are immune, however my experience indicates that the decay rate (i) varies dramatically from learner to learner and (ii) is much more dependent on the learner than the quality of teaching.



Kreuger's 1929 research tells us that although overlearning (practising beyond one successful solving of a problem type) does increase retention; for each extra practice question, the learner embeds that learning progressively less, until soon there is no further improvement.

Let's relate this to teaching 3 skills on a topic, let's call them A, B and C and assume that A is a pre-requisite to learn B and B is a pre-requisite to learn C. In a traditional units-of-related-topics, each-topic-once-a-year scheme of learning we might plan to teach one lesson on A, the second on B and the third on C. However what we know about overlearning means that in each lesson, the learners soon begin officially sanctioned wasting lesson time. This is shown in pattern (a); the following research summary builds up an argument for pattern (b)

Rohrer and Taylor's 2006, 2007 did two experiments finding out about the retention of "new to the student" maths learning of college students their results are summarised in the tables that follow.

Quantity of practice varied	**test 1 week later**	**test 4 weeks later**
3 practice problems	67%	27%
9 practice problems	69%	28%

It seems that (i) the extra practice problems that half the students did, made little appreciable difference to their retention and (ii) even for proven to be successful learners i.e. college students, within a month much new learning was forgotten.

Timing of practice varied	**1 week later 2007**	**4 weeks later 2006**
all the practice problems done in one session	75% ~~67%~~	32% ~~27%~~
half of the problems, wait a week, other half of the problems	70% ~~69%~~	64% ~~28%~~

It seems that a simple technique: splitting the practice questions into two sessions separated by a week, has a profound effect on retention. The gap of a week between practice sessions is not fixed in stone; if the teacher wishes to reduce re-teaching then the gap should be timed so that all the learners can recall their learning.

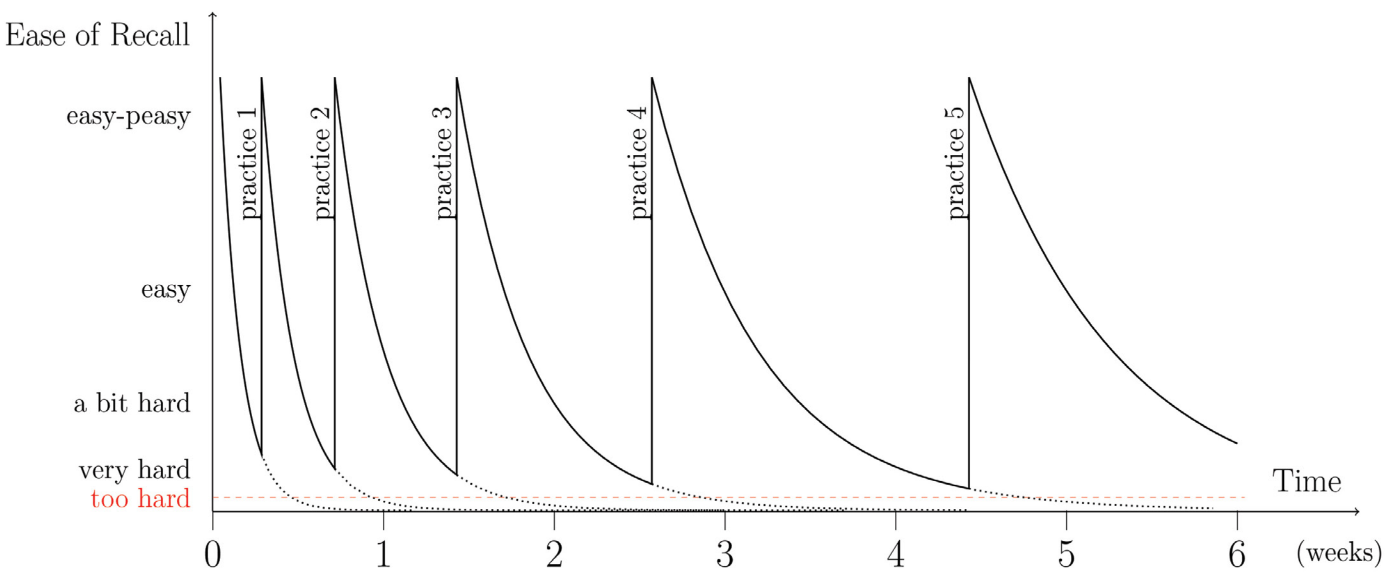
My experience says that most low attaining learners "appear to completely forget" new learning within three days; although when re-teaching the learners often re-learn quicker than last time, indicating that at least some learning has been retained.

My experience says that most low attaining learners "appear to completely forget" new learning within three days

Repeated retrieval practice with the spacing increased geometrically means that we can turn a "forgets within 3 days" learner to a "remembers for over a year" learner with a relatively modest number of practice questions.

When we teach a number of similar but different skills within a lesson Sweller's 2011 cognitive load theory tells us that (i) learners will **learn** better if we teach skill A and learners practise skill A, then teach skill B and learners practise skill B than if we teach skill A and B and then ask learners to practise skill A and B. (ii) Novice learners should practise problems very similar to the worked example. (iii) As learners become more experienced they can cope with more variety in the problems.

However once learners can remember what they have learned if we interleave different types of problems in a non predictable order, learners will be better able to **apply** their learning, than if they practice blocks of the same question type Rohrer 2009. The learner can no longer think "this is a lesson on A, so I must apply skills A to this problem" instead the learner must select the appropriate concept or procedure to match the features of the problem.



If learners are encouraged to (i) recall what they have learned, or (ii) "try before the teacher teaches" even if the learners are wrong, their memory will be enhanced, provided the teacher/text book/ video goes on to demonstrate a solution Kornell 2009. If the teacher teaches or reminds the learners before the learners have had a chance to try, this opportunity to deepen the learner's memory is lost.

feedback can only be useful if it can be retained by the learner until their next opportunity to apply it

The mechanism behind the "try first" improvement could be that the learners activate "useful for solving problems like this" parts of their memory, or that the "brain only bothers to strengthen connections when it feels it needs to". This links to Bjork's 2011 research in desirable difficulty: (i) we retain learning better if we struggle a little at the point of teaching. (ii) We embed learning deeper if we have to think hard to recall our learning, this is the mechanism that makes retrieval practice work. (iii) Until we are taught or otherwise find out, we tend to think if we are fluent in our practice that we are learning well and if we are struggling to apply our learning then we are not learning well. However a learner's fluent "learning performance" is not a reliable measure of learning, only time will tell if the new learning will become embedded or forgotten.

One "take away for teachers" is don't be discouraged if you ask a question and are met with "a sea of confused faces", or a selection of "off the wall suggestions". Instead say "now I've activated the relevant problem solving parts of your brain, it will be easier to learn …"

If we insist on sticking to an each-topic-once-a-year scheme of learning, pattern (b) will embed learning much better than pattern (a). In pattern (b) there is little or no lesson time spent on unproductive overlearning, blocked practice is split into two sessions. The second blocked practice lesson occurs after a "suitable for the class gap of time" and then after a longer gap of time a lesson of interleaved practice is done. The interleaved practice can replace an end of unit test if we so choose. Pattern (b) uses 3.5 lessons rather than the 3 lessons for pattern (a), however learners gain discrimination ability and much greater retention. Pattern (b) will work well for middle to high attaining learners. Before moving on to look at what else low attaining learners might need, I'd like to address the timing of assessment, feedback and working memory capacity.

In schools the review and assessment program generally suits high attaining learners: the timing of reviews e.g. homework, end of unit, mid year and end of year tests gives the highest attaining learners approximately "ideal for them" retrieval practice spacing. For low attaining learners, each test is too late, the learner must, if they are able, re-teach themselves everything in order "to revise".

We know that feedback is key to raising attainment Kluger 1996, especially for low attaining learners. However feedback can only be useful if it can be retained by the learner until their next opportunity to apply it.

I think most teachers of groups of low attaining learners will be familiar with the "after-test-results motivation slump" which can require the teacher to spend a number of lessons trying to boost the learners' interest and confidence so that learning can continue. This then is hardly an ideal time to feedback on unsuccessfully answered questions. Any post test review by the teacher also tends to be too concentrated, too much to learn at one time. What the teacher thinks of as "revision" often seems to the learners as "new learning".

mixing more than one problem type will overload the learner's working memory

My experience says (i) that review should happen when the learner can get about 80% of the questions correct and (ii) that a "test" can arouse a fight-or-flight response whereas retrieval practice, providing the teacher doesn't call it a test, does not.

It is estimated that 10% of learners would be diagnosed as having poor working memory, but most learners remain undiagnosed Gathercole 2008. Deliberate or retrieval practice over time enables "chunks Gobet 2005 " or "mental schema Sweller 2011 " to be formed in long term memory. These are able to replace the need for some "slots" in working memory. With suitable practice, over time, the learner will need less working memory to solve a problem as the chunks in long term memory can be used to take some of the working memory requirement.

learners with small working memories need no longer be disadvantaged by the curriculum

Having a smaller than average working memory can limit learners from successfully engaging in tasks which are designed to promote learning Gathercole 2008. A learner with a smaller working memory than their peers often can't function if "practice over time" of the pre-requisite skills has not enabled them to convert some of the working memory demands of the pre-requisite learning into long term memory chunks. So learners who most need to rely on long term memory, if we are not careful, are the very learners who are least likely to build up chunks in their long term memory.

Cognitive Load Theory Sweller 2011 tells us a lot about how to reduce un-necessary working memory demands. (i) Listening to the teacher read out what is written on a powerpoint requires more working memory than either listening or reading. (ii) Using example-problem pairs (a worked example and then a problem which is both very similar and laid out almost identically) has lower working memory demands than traditional teaching when introducing new learning, (iii) but learners attempting to use an unmatched example to answer a problem has a really high working memory load (iv) Example only study can be as effective as example-problem pairs, but since low attaining learners in particular are not very good at studying worked examples, completion problems (part worked example, part "and now finish this problem") can work better. Van Gog 2010.

Solving a "mixed bag" of problems is valuable in improving learners ability to apply their learning but if a learner needs all their working memory capacity to solve a single type of problem, then mixing more than one problem type will overload the learner's working memory.

Returning to the idea of teaching skills A, B and C on a topic, both patterns (a) and (b) will fit with an each-topic-once-a-year scheme of learning. If we take away the ubiquitous but somewhat arbitrary each-topic-once-a-year requirement, we could teach one skill per term, providing we also ensure that learning is embedded through retrieval practice. This is shown in pattern (c). The major advantage of pattern (c) is that learners with small working memories need no longer be disadvantaged by the curriculum.

I've tried pattern (c) with a class of low attaining learners and it works really well, in terms of engagement, behaviour, attendance and especially in terms of raising attainment. However it was incredibly time consuming, especially as the group was not homogenous, so necessary differentiation made pattern (c) that much harder to manage.

I've spent the last six years working with Simon Booth a software engineer to create timely practice to enable pattern (c) to be used. Initial results seem to support the step change increase in attainment of previously low attaining learners, whilst keeping the duration of the teacher's assessment + planning + resourcing similar to that of traditional teaching. An article about timely practice will be in the next equals magazine. In the meantime if what I've described "makes sense" or intrigues you then please visit www.timelypractice.com

Lesley Goddard

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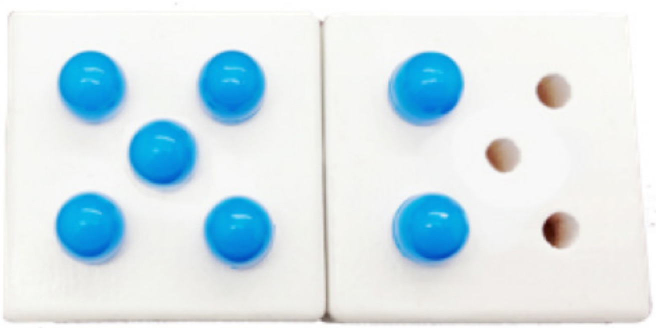
Spot on with Number

Carol Handyside **introduces a new resource aimed at supporting the early development of number sense.**

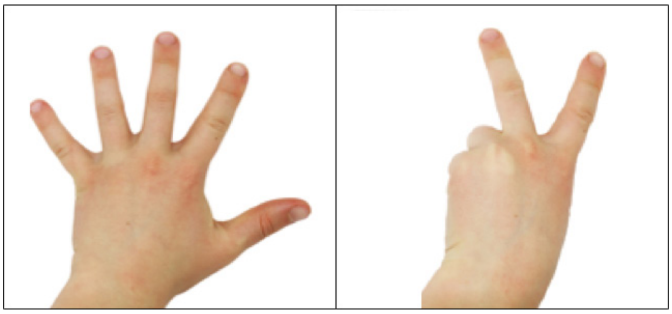
When asked to show me 7 fingers, Mia counted each of her fingers, one by one, until she had raised 7. I worked with Mia in a small intervention group for 10 minutes once a week in the final term of year one. The children worked with making numbers on a five dice pattern peg board and practiced number bonds to 10. Mia and the other children in the intervention group enjoyed the resources and would excitedly ask when they saw me arrive 'are we going to play the fun maths game?'

We weren't actually playing, the children were following a set of activities from the Spot On With Numbers Number Bonds Guide on making numbers and thinking about how numbers are made up. However, to the children, they were playing, as they were learning through making and exploring numbers, with questions and activities to guide their play.



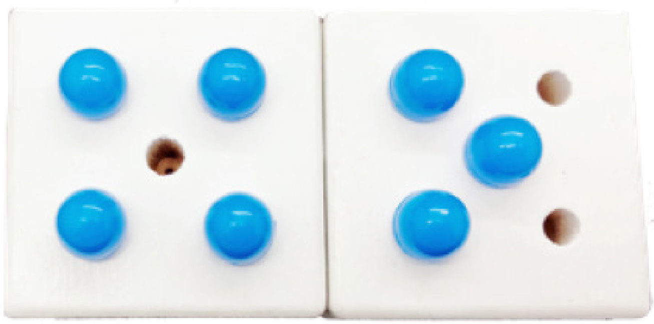
For example, the children were asked to make the number 7 by counting pegs of the same colour onto the board.

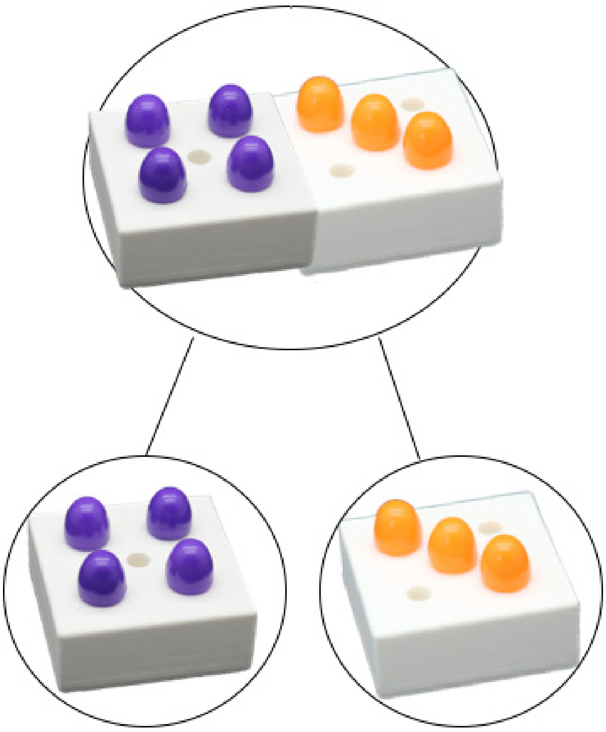
I then asked the children to show me the number in the same way with their fingers. This gave Mia a strong multisensory representation of 7 which she could use as a foundation.



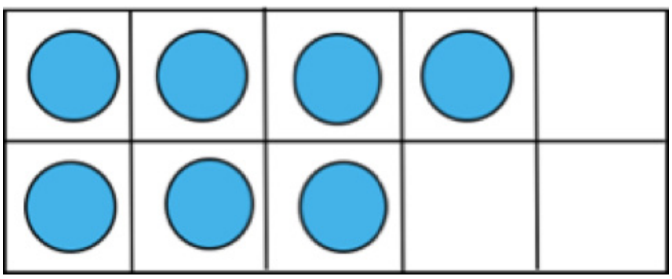
The fact that Mia relied heavily on counting was prohibiting her development of number sense. Without this intervention, she would most certainly be left behind as the size of the numbers she encountered in her maths lessons got larger, with the need to understand place value and master many more number facts.

It was necessary to first develop Mia's sense of the numbers within 10. In the example of 7 above, the pegs and boards and the child's fingers naturally decompose 7 into two parts. The children were asked to communicate this using language such as 'I can see that 5 add 2 equals 7'. By asking the children to move one peg from the left and add it onto the peg board on the right, the children could see the same number partitioned differently. The children were asked to explain the link, so that they were not learning a new fact, but using the fact they had already acquired to learn an additional one.

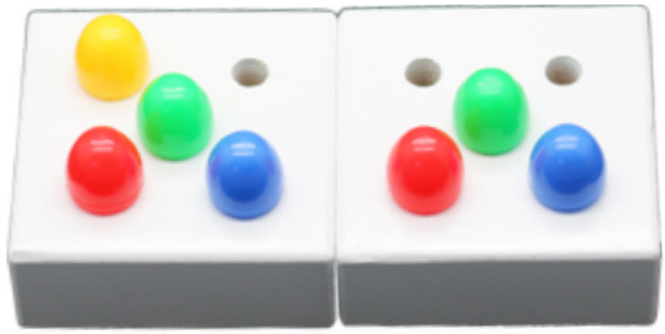


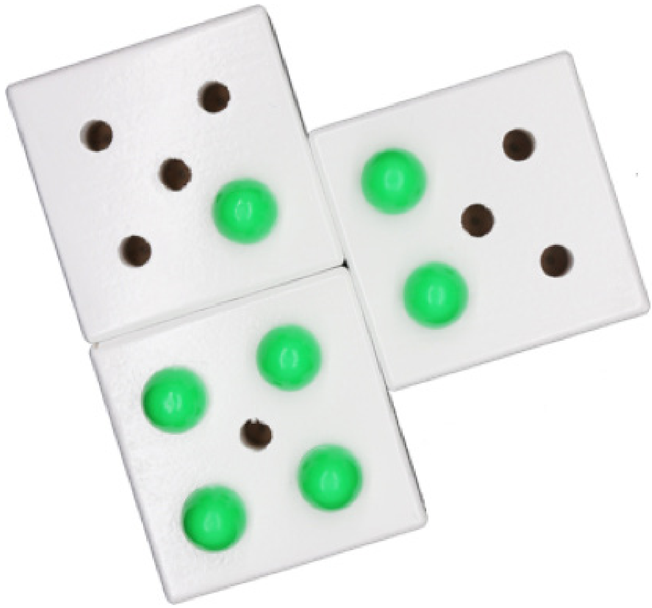


We also looked at different configurations of 7 with different colours to see how 7 was made up and, where possible, linked the configurations to our fingers. The children became confident at partitioning and were much more efficient at showing this on their fingers. We linked the pegs and boards not only to the fingers, but also to other resources in the classroom asking 'what is the same, what is different?'



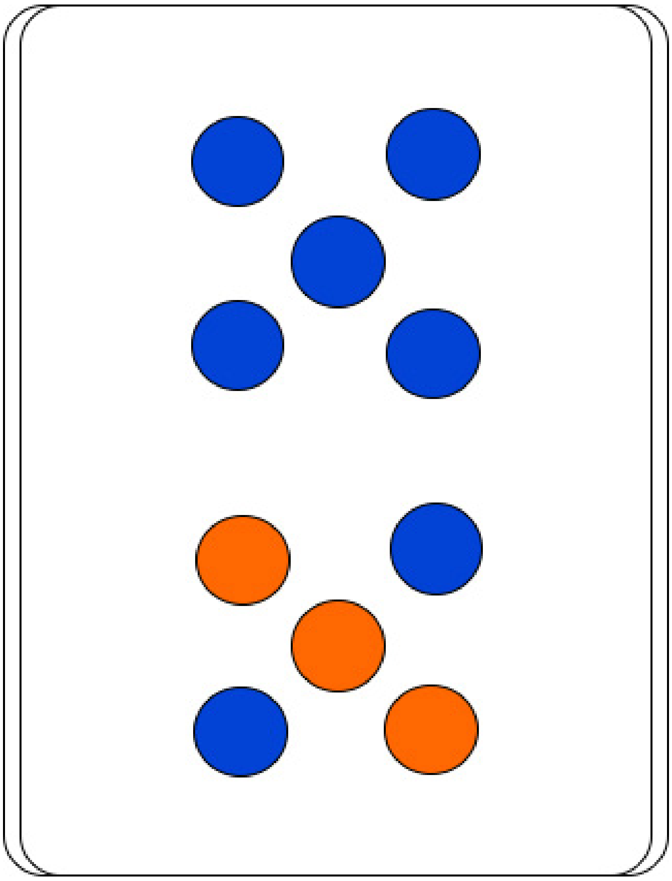
There are many more ways of making 7 with the pegs and boards which can be used to strengthen the ability to subitise, see connections and appreciate patterns, all of which will give a deeper sense of the number 7.



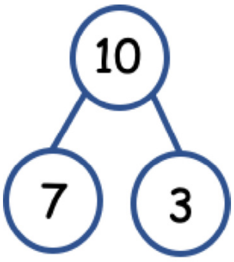


We repeated the activities for 6, 8 and 9.

After exploring the numbers less than 10, we looked at which two numbers added to a total of 10.

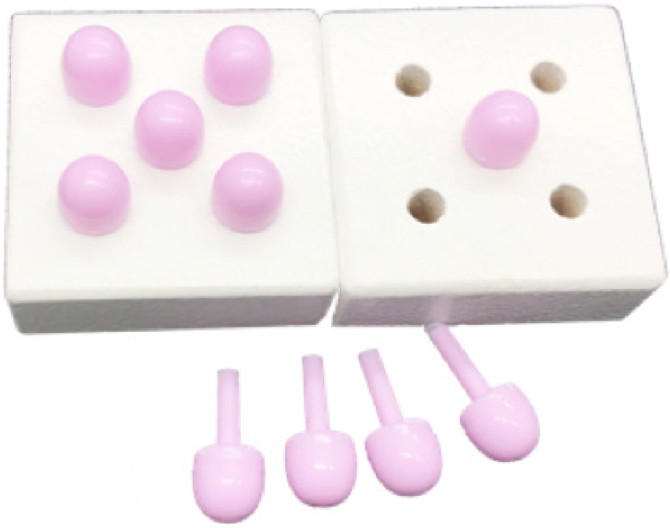


The group particularly enjoyed the transparent 'magic cards', where numbers from 0 - 10 could be joined together to show two numbers totalling ten. The children were encouraged to use the mathematical language they had learnt; '7 add 3 equals 10'. We explored the part-part whole model, commutativity and subtraction as the inverse of addition.

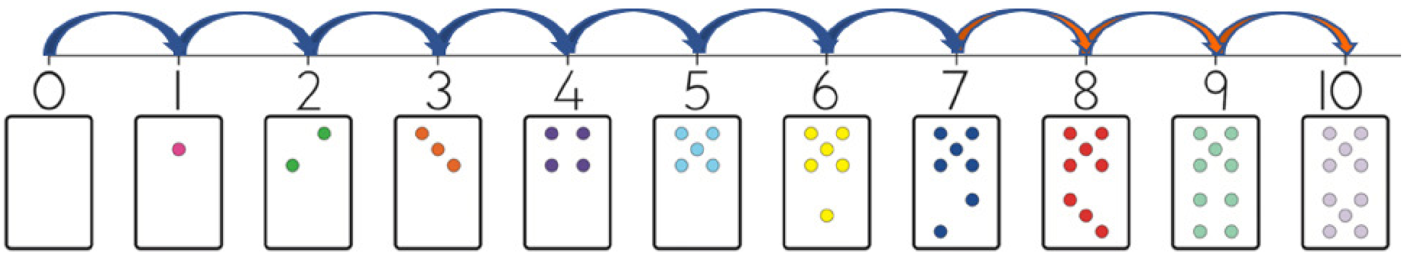


I could tell that Mia had developed a firmer understanding and better sense of the numbers within 10. She rarely counted in ones and when asked to show me, for instance 7 fingers, she could respond immediately. She quickly picked up her number bonds to 10 and began to use these facts flexibly.

The children initially struggled with questions like 10 + ___ = 10, but with both the pegs and boards and transparent cards, zero can be shown, so this was soon overcome. ☐ - 4 = 6 could be modelled with the pegs and boards too. The Spot On With Numbers resources can model all whole number sentences up to 100 and have many more applications, but as an initial launch, our focus is on number bonds.



The products have been promoted by specialists in dyscalculia as they offer the dice dot patterns that are encouraged and also link to the fingers. The breakdown of quantities encourages children to partition and the pegs and boards tie in with



Steve Chinn's 'Maths Explained' programme. There is a link between maths learning difficulties and a lack of ability to subitise, so the products offer a practical solution to promote and strengthen subitising skills. Dyslexic students, with strong visual or reasoning skills, who struggle to retain number facts will benefit from being taught using the products. Spot On With Numbers provides a different way of seeing numbers, so provides structural variation, which is useful for all children to gain a deeper understanding of number.

For more information, please visit https://www.spotonwithnumbers.co.uk where you will also find a free resource, so you can try out some of these activities: https://www.spotonwithnumbers.co.uk/free-resources.

Carol Handyside

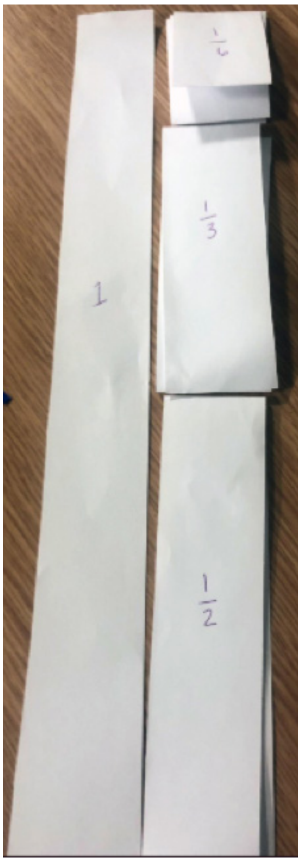
A review of the *Equals* presentation at the London MA branch meeting

Kirsty Behan reports on what was said by Pete and Alan at the Institute of Education on the 29th June.

The London joint MA/ATM branch meetings happen six times over the academic year, they are opportunity to share best practice and are led by either one or more professionals from maths education. The final session of the year was led by Alan, Pete and myself, coming from the perspective of supporting practitioners support those students who find learning maths most challenging. This article is a summary of the ideas shared on that morning.

Alan began by sharing a variety of activities that he has spent time working on that designed for students in primary school, that can also be used in key stage 3 nurture classes. The overriding theme of Alan's session was about providing opportunities for you as an educator to see where the children are coming from, and using that to support them in their mathematical development. The activities are designed as a starting point and can be used in a multitude of ways to elicit different parts of learning but have a common theme of trying to get students to talk about maths, to you as well as each other.

Alan presented four different activities that could be used as starting points for learners. The one



that resonated the most with me was using strips of paper as a starting to form fractions. We began with six strips of paper. The first left as a whole strip, the second we were asked to fold into 2, the third into 3, the fourth into 4, the third into 6 and the final strip into 8. We were then asked to label them as we see fit, this for me presented an ideal opportunity for you as a teacher to see what students' prior knowledge and understanding of fractions was as well as elicit misconceptions and demonstrate using a concrete representation why that might not be quite correct. To me, and to others, this then seemed to create a starting point for equivalence in fractions, comparing fractions as well as adding fractions. (Above is an image taken on the morning showing how they can be used to show an example of fractions adding to make 1). This is definitely something I am going to try next year with a nurture year 7 class, with the aim of continually referring back to the model and using them to begin different learning points.

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Alan also shared broken rulers and giants palace, which he shared in a previous *Equals* edition, which are two other favourites of mine that begin as starting points in measurement which, again, lead to interesting discussions and sticking points for students.

Throughout Alan's session the main thing that struck me was the importance of allowing students to share their own experiences and thoughts as a way to see how they view the world and, therefore leading to, how can I help them learn and adapt their thinking to be able to apply their view to mathematics. Secondary to this, sharing these activities as adults and educators still allowed this important discussion to take place. This shows me the importance of taking time, wherever possible in very busy teaching schedules, to sit down with colleagues and share resources, share ideas and have mathematical discussions.

The second half of the meeting was led by Pete who discussed that latest developments in defining dyscalculia as well as sharing some of the common difficulties learners who find maths more challenging have. The first thing that Pete shared, which really struck a chord with myself and the room, was about a survey that was conducted on how happy students were at school. The mean response for how they felt about school was 3.33, which is between neither happy or unhappy and happy, and the modal response was a 4, happy. However, when the same students were asked about their happiness in maths lesson the mean was 2.88 yet the mode was 1, with 29% of students saying they were very unhappy in a maths classroom. Whilst I am aware that there is often a stigma around mathematics, this is much higher than I would have thought. Further to this, Pete shared that, when researched in 2012, 24% of the adult population in UK are functionally innumerate which many people in the room seemed to be shocked by. But what is causing this?

Pete shares that between 5-7% of learners have persistent specific difficulties in maths. He also shared that in a study of 2421 primary schools that whilst 4.46% of students had an official diagnosis of Dyslexia only 1 student, 0.04%, had an official diagnosis of Dyscalculia. I think this is something important to be aware of as a mathematics educator, as whilst many specific learning difficulties can go undetected this appears to be even more so. Pete summarised that Dyscalculia is a core deficit in numerosity, having a sense of magnitude. It affects subitising (the ability to recognise a collection of objects, usually 5-7, without counting), symbolic and non-symbolic magnitude comparisons and ordering. It can occur on its own or co-occur with other learning difficulties.

Also shared, were the indicators of dyscalculia. Which include: an inability to subitise small quantities (as low as 2-3), poor number sense, magnitude processing difficulty, profound difficulty in estimating whether a numerical answer is reasonable, immature strategies, profound difficulty in noticing patterns, profound difficulty in generalising, directional confusion, slow processing speed, difficulty sequencing, difficulty in language, poor memory for facts and procedures, weaknesses in short and long term memory, inability to count backwards reliably, weakness in spatial and visual orientation, problems with all aspects of money, difficulties with word problems and multi-step calculations.

Other than specific learning difficulties it is also important to be aware of what else can impact on maths learning such as:

- Mindset and resilience
- Confidence and anxiety
- Literacy
- Reasoning
- Memory and Speed of processing
- Arithmetic and number sense – dyscalculia

Whilst the initial statistics in happiness in the maths classroom left me feeling somewhat disparaged, it is clear the issue is wide-ranging in the complexities that students can bring to the mathematics classroom. However, I think, by being aware of this complex picture and taking the time to reflect on how we can address each of these issues for the whole class, as well as looking at the individual, there is hope and opportunity to change the experiences that students can have in mathematics. For every individual, by analysing what are their challenges as well as using resources that help you to see their thinking, there is opportunity for success and for them to feel happy in the maths classroom.

If you would like any of the resources that were discussed Alan is willing to share them so please email him, and if you have any responses feel free to tweet us @Equalsonline. We are aiming to hold another day similar to this one in the future so we hope that more people can become involved and more ideas can be shared, keep an eye on the coming issues as well as twitter.

Kirsty Behan